
Collision Avoidance for Multi-Vehicle Systems

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**Happy Birthday Mark
and thank you for hiring me
(hope this is not one of the
reasons why you left Illinois)!**

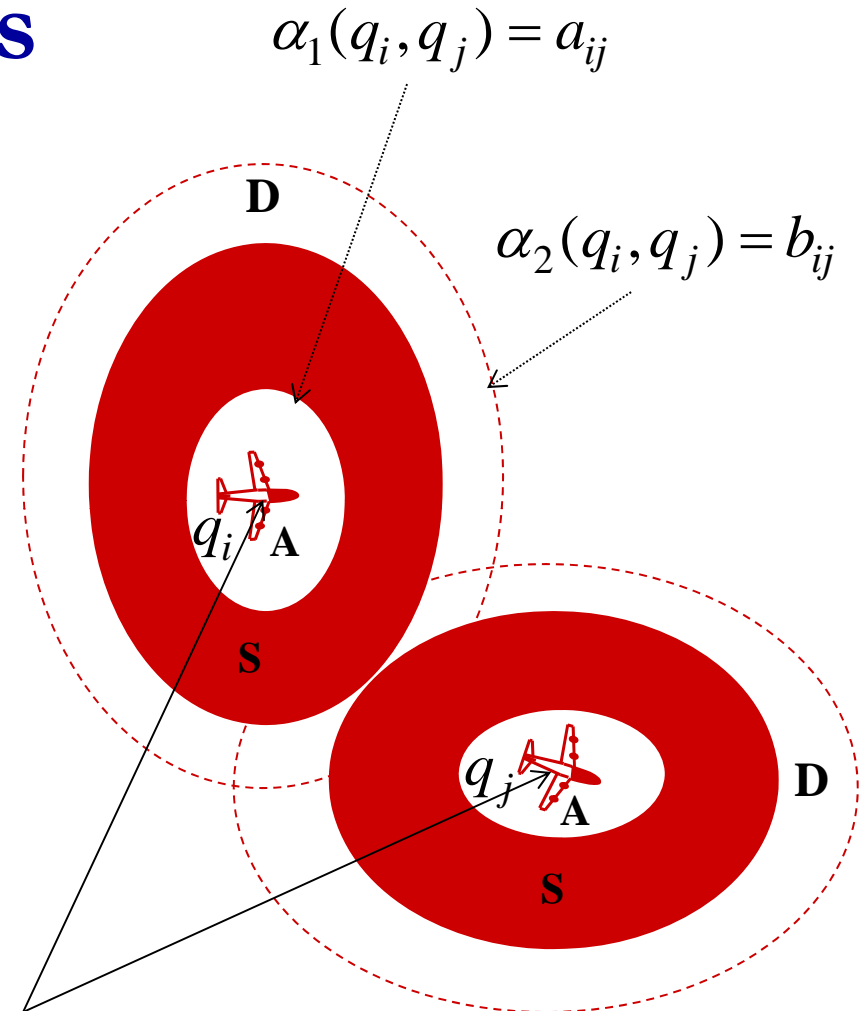
Avoidance Functions

“Quadratic” Avoidance Functions

$$v_i = \sum_{j \in \mathbb{N}} v_{ij}, \quad v_{ii} = v_{ii}^o$$

$$v_{ij} = \left(\min \left\{ 0, \frac{\alpha_2(q_i, q_j) - b_{ij}}{\alpha_1(q_i, q_j) - a_{ij}} \right\} \right)^2, \quad i \neq j$$

$$\alpha_k(q_i, q_j) = (q_i - q_j) P_k (q_i - q_j)^T, \quad k = 1, 2$$



Cooperative Collision Avoidance for Co-axial Helicopters

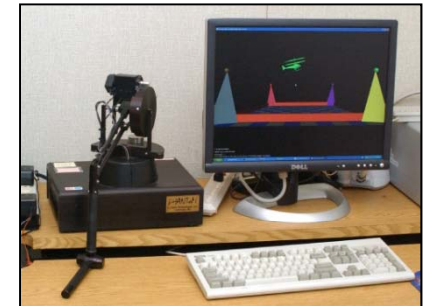
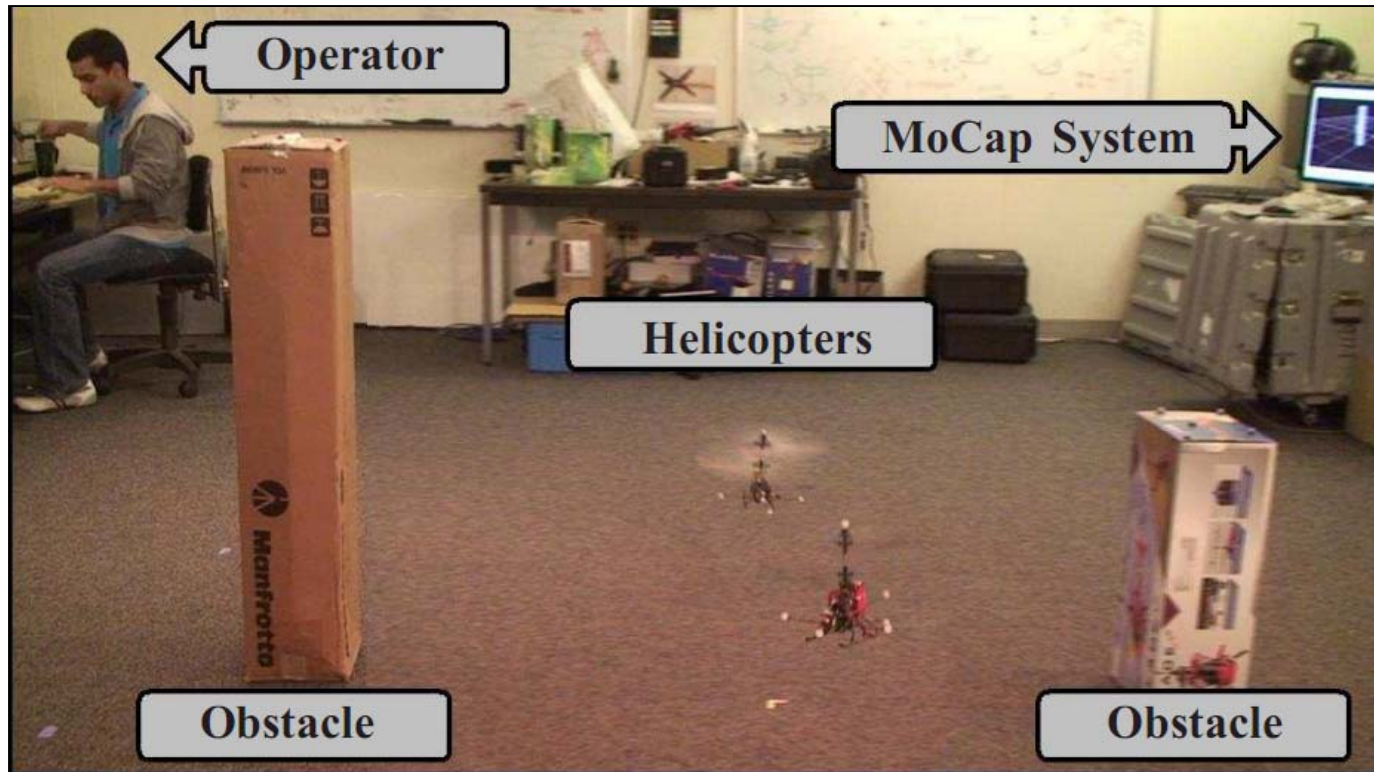


Cooperative Collision Avoidance

Tactical Robot Teams Project

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Bilateral Teleoperation and Collision Avoidance



Master Robot



Slave Vehicles

Coordination:

- Trajectory Tracking
- Formation Control

Safety:

- Stability under Communication Delays
- Collision Avoidance

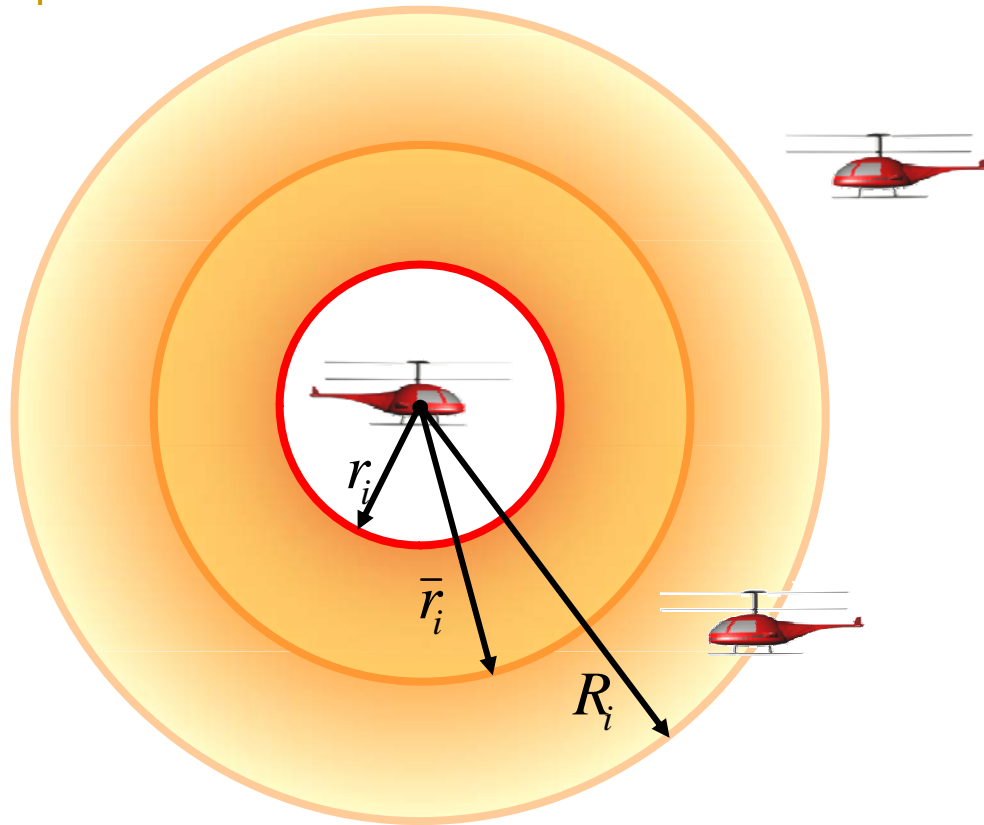
Transparency:

- Operator must feel environmental forces interacting with remote agents.

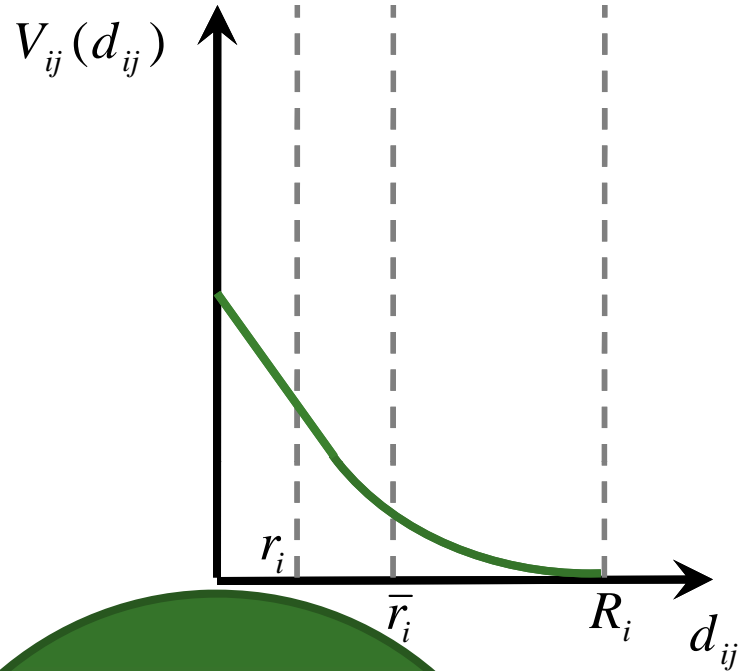
Collision Avoidance with Detection Errors

Problem:

To guarantee safe navigation of mobile vehicles through obstructed and shared environments when the information is not perfect.



Bounded Avoidance Control



Avoidance Functions

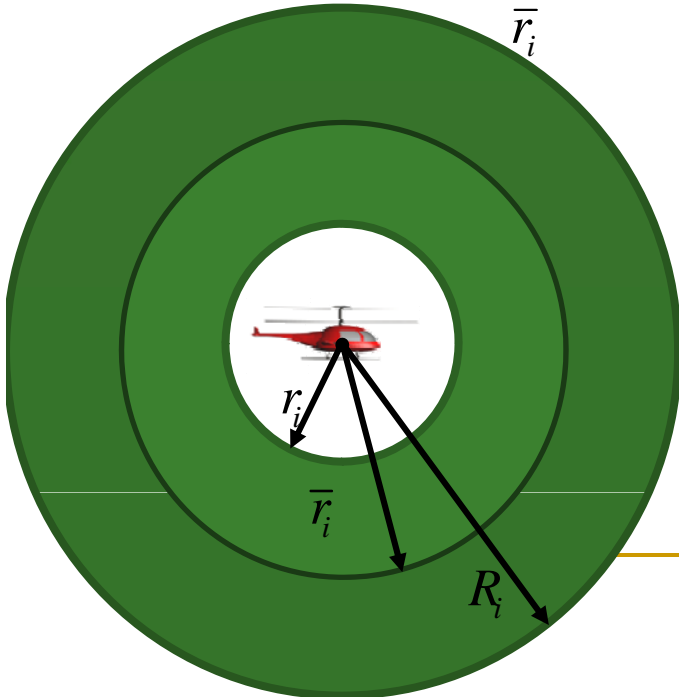
$$V_{ij}(d_{ij}) = \begin{cases} \alpha_i \left(\min \left\{ 0, \frac{d_{ij}^2 - R_i^2}{d_{ij}^2 - r_i^2} \right\} \right)^2, & \text{if } \bar{r}_i \leq d_{ij} \\ -\mu_i \|d_{ij}\| + c_i, & \text{if } 0 < d_{ij} < \bar{r}_i \end{cases}$$

$$d_{ij}(t) = \|q_i(t) - \hat{q}_j(t)\|$$

$$\hat{q}_j(t) = q_j(t) + \varepsilon_j(t), \quad \|\varepsilon_j(t)\| \leq \Delta_j$$

Avoidance Control Component

$$u^a = - \sum_j \frac{\partial V_{ij}}{\partial q_i}$$



Collision Avoidance with Detection Delays

Three Coaxial Helicopters

$$T_{12} = 0.30s, T_{13} = 0.24s$$

$$T_{21} = 0.30s, T_{23} = 0.27s$$

$$T_{31} = 0.30s, T_{32} = 0.30s$$

$$v_1 = v_2 = v_3 = 1m/s$$

$$R = 1.00m, r = 0.25m$$

Collision Avoidance with Sensing Uncertainties: Avoiding Singular Points



Objective
Control

Avoidance
Control

Control Law: $u_i = u_i^o + u_i^a + u_i^\varepsilon - k_i \dot{q}_i$

Perturbation (vector)
perpendicular to the avoidance
control when the i -the agent
approaches an unwanted
local minima

Maximum Velocity
Regulation

Singular Points: $u_i^o \rightarrow -\lambda u_i^a, \lambda > 0$

Collision Avoidance with Sensing Uncertainties: Simulation Results

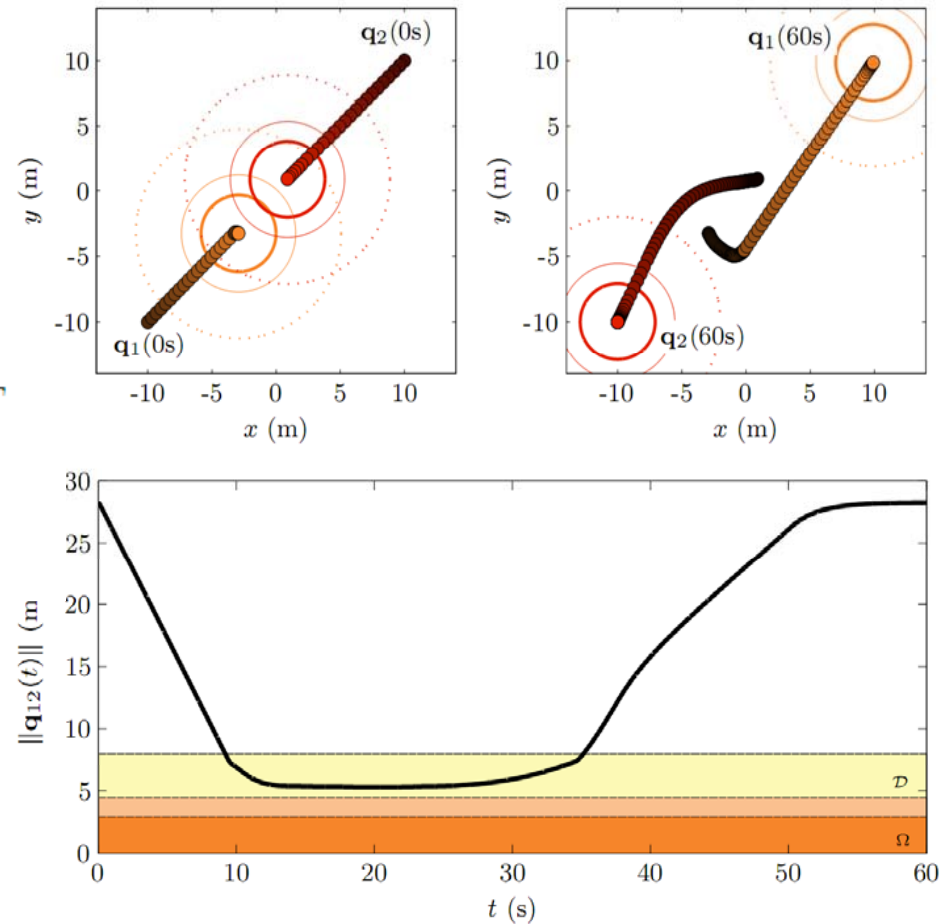
Sensing Uncertainty (Error):

$$\mathbf{d}_i(\mathbf{q}_i, \mathbf{q}_j) = \zeta_{qi}(\mathbf{q}_i, \mathbf{q}_j) + \zeta_{ei}(\mathbf{q}_i)$$

$$\zeta_{qi}(\mathbf{q}_i, \mathbf{q}_j) = a_{1i} \left[\left[\frac{q_{j1} - q_{i1}}{a_{2i}} \right], \left[\frac{q_{j2} - q_{i2}}{a_{2i}} \right] \right]^T$$

$$\zeta_{ei}(\mathbf{q}_i) = a_{3i} [\cos(q_{i1}), \sin(q_{i2})]^T$$

- First Agent, $\Delta_1 = 1.0\text{m}$
- Second Agent, $\Delta_2 = 0.5\text{m}$



Questions?

